Torsión. Planteamiento en desplazamientos:

Barra cilíndrica de directriz recta, sección transversal de forma cualquiera, llena y constante.

Sistema de referencia: *origen* : *centro*, OX_3 : *eje axial* .

Se somete en sus extremos a sendos giros $-\alpha_0$ y α_0, alrededor del eje axial.

Hipótesis de deformación de Saint-Venant (observación experimental):

1.- Las secciones transversales solo giran como sólidos rígidos alrededor del eje z, es decir, no se deforman ni distorsionan en un plano.

El ángulo α de giro de cada sección varía linealmente entre los ángulos de giro de las secciones extremas:

$$\alpha = \theta x_3$$

siendo θ : giro por unidad de longitud.

2.- Las fibras de la dirección Ox_3 no se deforman, es decir:

$$\varepsilon_{33} = 0 \Rightarrow u_3 = u_3(x_1, x_2)$$

Torsión. Planteamiento en desplazamientos:

$$\begin{cases} u_1 = -\theta x_2 x_3 \\ u_2 = \theta x_1 x_3 \\ u_3 = \theta f(x_1, x_2) \end{cases}$$

Ecuaciones de Navier – Cauchy :

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} + \rho b_i = 0 \Rightarrow \nabla^2 f = 0$$

$$\nabla^2 f = 0 \qquad \text{(} u_{j,ji} = 0,\ \rho b_i = 0 \text{)}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) = \begin{pmatrix} 0 & 0 & \frac{\theta}{2}\left(-x_2 + \frac{\partial f}{\partial x_1}\right) \\ & 0 & \frac{\theta}{2}\left(x_1 + \frac{\partial f}{\partial x_2}\right) \\ & & 0 \end{pmatrix}$$

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda e \delta_{ij} = \begin{pmatrix} 0 & 0 & \mu\theta\left(-x_2 + \frac{\partial f}{\partial x_1}\right) \\ & 0 & \mu\theta\left(x_1 + \frac{\partial f}{\partial x_2}\right) \\ & & 0 \end{pmatrix}$$

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orsión. Planteamiento en desplazamientos:

Condiciones de contorno en el contorno lateral:

$$t_i = \sigma_{ji} l_j = \begin{pmatrix} 0 & 0 & \mu\theta\left(-x_2 + \frac{\partial f}{\partial x_1}\right) \\ 0 & 0 & \mu\theta\left(x_1 + \frac{\partial f}{\partial x_2}\right) \\ \mu\theta\left(-x_2 + \frac{\partial f}{\partial x_1}\right) & \mu\theta\left(x_1 + \frac{\partial f}{\partial x_2}\right) & 0 \end{pmatrix} \begin{pmatrix} l_1 \\ l_2 \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ 0 \\ l_1 \mu\theta\left(-x_2 + \frac{\partial f}{\partial x_1}\right) + l_2 \mu\theta\left(x_1 + \frac{\partial f}{\partial x_2}\right) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$l_1 \mu\theta\left(-x_2 + \frac{\partial f}{\partial x_1}\right) + l_2 \mu\theta\left(x_1 + \frac{\partial f}{\partial x_2}\right) = 0 \Rightarrow$$

$$l_1\left(-x_2 + \frac{\partial f}{\partial x_1}\right) + l_2\left(x_1 + \frac{\partial f}{\partial x_2}\right) = 0 \Rightarrow \mathbf{n} \bullet \left(\nabla f + \left(\, -x_2 ,\, x_1 \,\right)\right) = 0$$

$$\frac{\partial f}{\partial \mathbf{n}} = \mathbf{n} \bullet \left(\, x_2 \quad -x_1 \,\right)$$

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Torsión. Planteamiento en desplazamientos:

$$\begin{cases} \nabla^2 f = 0 & en\,\Omega \\ \frac{\partial f}{\partial \mathbf{n}} = \mathbf{n} \bullet \left(\, x_2 \quad -x_1 \,\right) & en\,\Gamma \end{cases}$$

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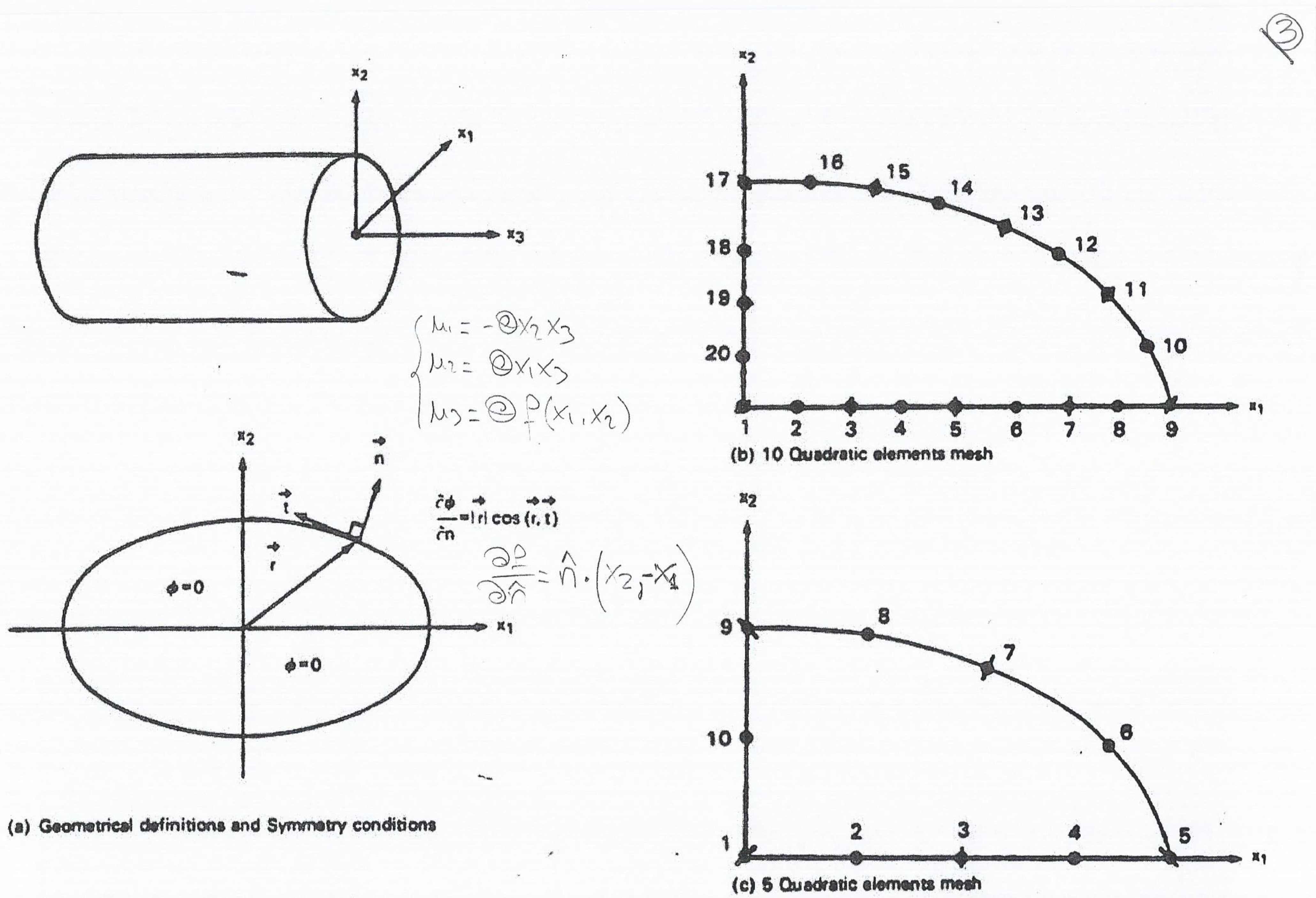

Figure 2.17 Torsion problem. Definition of the problem and boundary discretizations

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PRUEBA.DAT

```
ELLIPTICAL SECTION UNDER TORSION (10 QUADRATIC ELEMENTS)
10  2
0. 0. 1.25 0. 2.5 0. 3.75 0. 5. 0. 6.25 0. 7.5 0. 8.75 0. 10. 0.
9.67 1.273  8.814  2.3617 7.7008 3.1898 6.174 3.933
4.7898  4.3891 3.3044 4.719 1.557 4.939
0. 5. 0. 3.375 0. 2.5  0. 1.25
0 0. 0 0. 0 0.
 0 0. 0 0. 0 0.
 0 0. 0 0. 0 0.
 0 0. 0 0. 0 0.
1 0. 1 -3.379  1 -4.8334
1 -4.8334 1 -4.9447  1 -4.3104
1 -4.3104 1 -3.4657  1 -2.4411
1 -2.4411 1 -1.1643  1 0.
0 0. 0 0. 0 0.
 0 0. 0 0. 0 0.
2. 2. 4. 3.5
```

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prueba10

****

ELLIPTICAL SECTION UNDER TORSION (10 QUADRATIC ELEMENTS)

DATA
NUMBER OF BOUNDARY ELEMENTS= 10
NUMBER OF INTERNAL POINTS= 2

BOUNDARY NODES COORDINATES

NODE	X	Y
1	.0000000E+00	.0000000E+00
2	.1250000E+01	.0000000E+00
3	.2500000E+01	.0000000E+00
4	.3750000E+01	.0000000E+00
5	.5000000E+01	.0000000E+00
6	.6250000E+01	.0000000E+00
7	.7500000E+01	.0000000E+00
8	.8750000E+01	.0000000E+00
9	.1000000E+02	.0000000E+00
10	.9670000E+01	.1273000E+01
11	.8814000E+01	.2361700E+01
12	.7700800E+01	.3189800E+01
13	.6174000E+01	.3933000E+01
14	.4789800E+01	.4389100E+01
15	.3304400E+01	.4719000E+01
16	.1557000E+01	.4939000E+01
17	.0000000E+00	.5000000E+01
18	.0000000E+00	.3375000E+01
19	.0000000E+00	.2500000E+01
20	.0000000E+00	.1250000E+01

BOUNDARY CONDITIONS

	-----FIRST NODE------		-----SECOND NODE-----		-----THIRD NODE------	
ELEMENT	PRESCRIBED VALUE	CODE	PRESCRIBED VALUE	CODE	PRESCRIBED VALUE	CODE
1	.0000000E+00	0	.0000000E+00	0	.0000000E+00	0
2	.0000000E+00	0	.0000000E+00	0	.0000000E+00	0
3	.0000000E+00	0	.0000000E+00	0	.0000000E+00	0
4	.0000000E+00	0	.0000000E+00	0	.0000000E+00	0
5	.0000000E+00	1	-.3379000E+01	1	-.4833400E+01	1
6	-.4833400E+01	1	-.4944700E+01	1	-.4310400E+01	1
7	-.4310400E+01	1	-.3465700E+01	1	-.2441100E+01	1
8	-.2441100E+01	1	-.1164300E+01	1	.0000000E+00	1
9	.0000000E+00	0	.0000000E+00	0	.0000000E+00	0
10	.0000000E+00	0	.0000000E+00	0	.0000000E+00	0

****

RESULTS

BOUNDARY NODES

X	Y	POTENTIAL	POTENTIAL DERIVATIVE BEFORE NODE	POTENTIAL DERIVATIVE AFTER NODE
.00000E+00	.00000E+00	.00000E+00	-.26532E-03	-.26532E-03
.12500E+01	.00000E+00	.00000E+00	.74965E+00	.74965E+00
.25000E+01	.00000E+00	.00000E+00	.14996E+01	.14996E+01
.37500E+01	.00000E+00	.00000E+00	.22502E+01	.22502E+01
.50000E+01	.00000E+00	.00000E+00	.29999E+01	.29999E+01
.62500E+01	.00000E+00	.00000E+00	.37536E+01	.37536E+01
.75000E+01	.00000E+00	.00000E+00	.44914E+01	.44914E+01
.87500E+01	.00000E+00	.00000E+00	.53030E+01	.53030E+01

ESTRUCTURA DEL FICHERO DE DATOS:

```
!
@AUX.PTO
!
'viga en voladizo. Pag 262 libro Pepón'

0.2 80000 4 1 1

 5  1 'C 0.0'  1 'C 0.0'  'LIN'
  0.0 0.0   20.0 0.0  1.
 1  1 'C 0.0'  1 'Q 0.0 1500.0 0.0 0.5'  'LIN'
  20.0 0.0   20.0 4.0 1.
 5  1 'C 0.0'  1 'C 0.0'  'LIN'
  20.0 4.0   0.0 4.0 1.
 1  0 'C 0.0'   1 'Q 0.0 -1500.0 0.0 0.5'  'LIN'
  0.0 4.0   0.0 0.0 1.

1 T T

10 2 0 0 0 0
```